PAPER – II : MODEL PAPER – 07

(SPECIMEN PAPER)

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. The ratio of prices of two cycles was 16:23 . Two years later when the price of first cycle has increased by 10% and that of second by ₹ 477 ; the ratio of prices becomes 11 : 20 . Find the original prices of two cycles

SOLUTION

Let price of first cycle = 16x

Price of second cycle = 23x

As per the given condition

$$\frac{16x + \frac{10}{100}(16x)}{23x + 477} = \frac{11}{20}$$

$$16x + \frac{16x}{10} = \frac{11}{20}(23x + 477)$$

$$\frac{176x}{10} = \frac{11}{20}(23x + 477)$$

$$352x = 253x + 5247$$

$$99x = 5247$$

$$x = 53$$

Price of first cycle = 16x = 16(53)= ₹ 848

Price of second cycle = 23x = 23(53)= ₹ 1219

02. for an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is ₹ 10,000 . What is the accumulated value after 3 years

($1.1^3 = 1.331$)

SOLUTION

P = □ 10,000 , n = 3 ; i = 0.1

$A = P(1 + i)^n$

$= 10000((1 + 0.1)3$

$= 10000(1.331)$

= ₹ 13,310

03. the probability of defective bolts in a workshop is 40% . Find the mean and variance of defective bolts out os 10 bolts

SOLUTION

n = 10 ,

p = prob of a defective bolt $= \frac{40}{100} = \frac{2}{5}$

$q = 1 - p = \frac{3}{5}$

mean = np $= 10 \times \frac{2}{5} = 4$

variance = npq $= 10 \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{5}$

04. a cargo of rice is insured at 5/8% to cover 80% its value . The premium paid is □ 5,250 . If the rice is worth ₹ 21 per kilo , how many kilos of rice did the cargo contain

SOLUTION

value of carge = x

insured value $= \frac{80}{100}x = \frac{4x}{5}$

Rate of premium $= \frac{5}{8}\%$

Premium = ₹ 5,250

$$5250 = \frac{5}{800} \times \frac{4x}{5}$$

x = ₹ 10,50,000

value of cargo = □ 10,50,000

price of rice = □ 21 per kilo

no of kgs of rice in the cargo $= \frac{10,50,000}{21}$

= 50,000 kg

05. the pdf of continuous random variable X is given by

$$f(x) = 2x \quad ; \quad 0 < x < 1$$
$$= 0 \quad ; \quad \text{otherwise}$$

SOLUTION

$P(1/3 < X < 1/2)$

$$= \int_{1/3}^{1/2} 2x \, dx$$

$$= \left[\frac{2x^2}{2}\right]_{1/3}^{1/2}$$

$$= \left[x^2\right]_{1/3}^{1/2}$$

$$= \left(\frac{1}{4}\right) - \left(\frac{1}{9}\right)$$

$$= \frac{5}{36}$$

06. an agent was paid □ 58,500 as commission on the sale of computers at the rate of 12.5% . If the price of each computer was ₹ 18,000 , how many computers did he sell

SOLUTION

Price of computer = ₹ 18,000

Rate of commission = 12.5%

Commission $= \frac{125}{1000} \times 18000$

$=$ ₹ 2250

Agents total commission = ₹ 58,500

Hence

No of computers sold $= \frac{58500}{2250} = 26$

07. $P(x) = \frac{x-1}{3} \quad ; \quad x = 1, 2, 3$

$= 0 \quad ; \quad \text{otherwise}$

Verify whether the function is a p.m.f.

SOLUTION

$P(x) = \frac{x-1}{3} \; ; \quad x = 1, 2, 3$

$p(1) = 0$
$p(2) = 1/3$
$p(3) = 2/3$

a) $p(x) \geq 0 \quad \forall x$

b) $\Sigma p(x) = 1$ Hence function is a pmf

08. if $X \sim N(4, 25)$; then find $P(x \leq 4)$

SOLUTION

$X \sim N(4, 25)$

In general

$X \sim N(\mu, \sigma^2) \quad \therefore \mu = 4$

In normal distribution ;

$P(x \geq \mu) = P(x \leq \mu) = 0.5$

Hence $P(x \leq 4) = 0.5$

Q5.

(A) Attempt any TWO of the following (06)

10. John and Mathew started a business with their capitals in the ratio 8 : 5 . After 8 months , John added 25% of his earlier capital as further investment . At the same time , Mathew withdrew 20% of his earlier capital . At the end of the year , they earned ₹ 52,000 as profit . How should they divide the profit between them

SOLUTION

PARTNER's NAME	CAPITAL INVESTED	PERIOD OF INVESTMENT
P	₹ 8k	8 MONTHS
+ 25%	₹ 2k	
	₹ 10k	4 MONTHS
Q	₹ 5k	8 MONTHS
−20%	₹ k	
	₹ 4k	4 MONTHS

STEP 1 :

Profits will be shared in the

'RATIO OF PRODUCT OF CAPITAL INVESTED & PERIOD OF INVESTMENT'

	P	:	Q	
=	8k x 8 + 10k x 4	:	5k x 8 + 4k x 4	
=	64k + 40k	:	40k + 16k	
=	104k	:	56k	
=	13	:	7	TOTAL = 20

STEP 2 :

Total profit = ₹ 52,000

P's share of profit $= \frac{13}{20} \times 52000$

$=$ ₹ 33,800

Q's share of profit $= \frac{7}{20} \times 52,000$

$=$ ₹ 19,200

02.

Age x	0	1	2
lx	1000	880	876
Tx	…..	…..	3323

Calculate e_0^0 , e_1^0 , e_2^0

SOLUTION

$$L_x = \frac{l_x + l_{x+1}}{2}$$

✓ $L_0 = \frac{l_0 + l_1}{2} = \frac{1000 + 880}{2} = 940$

✓ $L_1 = \frac{l_1 + l_2}{2} = \frac{880 + 876}{2} = 878$

$$T_{x+1} = T_x - L_x$$

✓ $T_2 = T_1 - L_1$

$3323 = T_1 - 878$

$T_1 = 4201$

✓ $T_1 = T_0 - L_0$

$4201 = T_0 - 940$

$T_0 = 5141$

$$e_x^0 = \frac{T_x}{l_x}$$

$e_0^0 = \frac{T_0}{l_0} = \frac{5141}{1000} = 5.141$

$e_1^0 = \frac{T_1}{l_1} = \frac{4201}{880} = 4.774$ (USE LOG)

$e_2^0 = \frac{T_2}{l_2} = \frac{3323}{876} = 3.793$ (USE LOG)

03.

in a town,10 accidents take place in a span of 50 days. Assuming that the number of accidents follow Poisson Distribution , find the probability that there will be one or more accidents per day $(e^{-0.2} = 0.8187)$

SOLUTION

m = average no of accidents per day

$= {}^{10}/_{50}$

$= 0.2$

r.v.x = no. of accidents per day

$X \sim P(m = 0.2)$

P(one or more accidents per day)

$= P(x \geq 1)$

$= P(1) + P(2) + \ldots\ldots\ldots.$

$= 1 - P(0)$

$= 1 - \dfrac{e^{-0.2}0.2^0}{0!}$ $\quad P(X) = \dfrac{e^{-m}m^x}{x!}$

$= 1 - e^{-0.2}$

$= 1 - 0.8187$

$= 0.1813$

Q5.

(B) Attempt any TWO of the following (06)

01. if the difference between true discount and bankers discount on a sum due 4 months hence is ₹ 20 , find true discount , bankers discount and the amount of the bill , the rate of simple interest charged being 5% p.a.

SOLUTION

STEP 1 :

BD – TD = 20

BG = 20

Int on TD for 4 months @ 5% = 20

$TD \times \dfrac{4}{12} \times \dfrac{5}{100} = 20$

$TD = \dfrac{20 \times 12 \times 100}{4 \times 5}$

TD = ₹ 1200

STEP 2 :

BD – TD = 20

BD – 1200 = 20

BD = ₹ 1220

STEP 3 :

BD = Int on FV for 4 months @ 5% p.a

$1200 = FV \times \dfrac{4}{12} \times \dfrac{5}{100}$

$FV = \dfrac{1200 \times 12 \times 100}{4 \times 5}$

FV = ₹ 72,000

02

X : 21 25 26 24 19

Y : 19 20 24 21 16

Obtain regression line X on Y

SOLUTION

x	y	$x - \bar{x}$	$y - \bar{y}$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
21	19	–2	–1	1	2
25	20	2	0	0	0
26	24	3	4	16	12
24	21	1	1	1	1
19	16	–4	–4	16	16
$\bar{x} = 23$	$\bar{y} = 20$	0		34	31

$bxy = \dfrac{\Sigma(x - \bar{x})(y - \bar{y})}{\Sigma(y - y)^2}$

$= \dfrac{31}{34}$

= 0.91 (DIRECT DIVISION)

$x - \bar{x} = bxy(y - \bar{y})$

$x - 23 = 0.91(y - 20)$

$x - 23 = 0.91y - 18.2$

$x = 0.91y + 4.8$

03. Determine the optimal sequence involving 5 jobs and three machines M_1 , M_2 and M_3 . The jobs are processed on three machines in the order $M_1M_2M_3$. Also find the minimum total elapsed time T and idle time for three machines . Processing time in minutes are

Job	J_1	J_2	J_3	J_4	J_5
M_1	7	12	11	9	8
M_2	8	9	5	6	7
M_3	11	13	9	10	14

STEP 1 : Min time on M_1 = 7 ; Max time on M_2 = 9 ; Min time on M_3 = 9

Min (M_3) $\geq$ Max (M_2) .... condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = M_1 + M_2

H = M_2 + M_3

Job	J_1	J_2	J_3	J_4	J_5
G	15	21	16	15	15
H	19	22	14	16	21

STEP 3 : OPTIMAL SEQUENCE

J_1	J_4	J_5	J_2	J_3

STEP 4 : WORK TABLE

Job	J_1	J_4	J_5	J_2	J_3		total processing time
M_1	7	9	8	12	11	=	47 min
M_2	8	6	7	9	5	=	35 min
M_3	11	10	14	13	9	=	57 min

JOBS	M_1		IDLE	M_2		IDLE	M_3		IDLE
	IN	OUT	TIME	IN	OUT	TIME	IN	OUT	TIME
						7			15
J_1	0	7	--	7	15	1	15	26	--
J_4	7	16	--	16	22	2	26	36	--
J_5	16	24	--	24	31	5	36	50	--
J_2	24	36	--	36	45	2	50	63	--
J_3	36	47	25	47	52	20	63	72	--

STEP 5 : Total elapsed time T = 72 min

Idle time on M_1 = T – (sum of processing time of all 5 jobs on M1) = 72 – 47 = 25 min

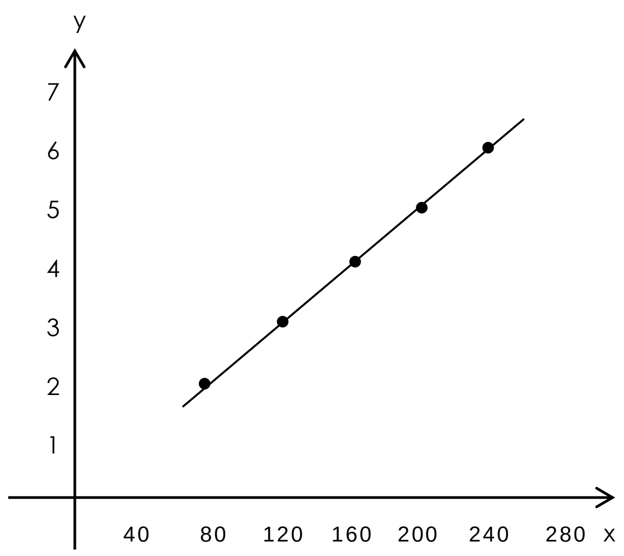
Idle time on M_2 = T – (sum of processing time of all 5 jobs on M2) = 72 – 35 = 37min

Idle time on M_3 = T – (sum of processing time of all 5 jobs on M3) = 72 – 57 = 15 min

Q6.

(A) Attempt any TWO of the following (06)

Distance (km)	**x**	80	120	160	200	240
Time (Hr)	**y**	2	3	4	5	6



COMMENT :

there is perfect positive correlation between the two variables

02.

for 20 pairs of observations on two variables x and y , the following data is available

$\Sigma(x-10) = 60$; $\Sigma(y-15) = 80$; $\Sigma(x-10)^2 = 990$; $\Sigma(y-15)^2 = 960$, $\Sigma(x-10)(y-15) = 480$

Find the correlation coefficient between x and y

SOLUTION

Let x – 10 = u & y – 15 = v

Given

$\Sigma u = 60$, $\Sigma v = 80$, $\Sigma u^2 = 990$, $\Sigma v^2 = 960$

$\Sigma uv = 480$, n = 20

$$r = \frac{n\Sigma uv - \Sigma u.\Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v^2)}}$$

$$= \frac{20(480) - (60)(80)}{\sqrt{20(990) - (60)^2}\sqrt{20(960) - (80)^2}}$$

$$= \frac{9600 - 4800}{\sqrt{19800 - 3600}\sqrt{19200 - 6400}}$$

$$= \frac{4800}{\sqrt{16200}\sqrt{12800}}$$

$$= \frac{48}{\sqrt{162}\sqrt{128}}$$

$$= \frac{48}{\sqrt{81 \times 256}}$$

$$= \frac{48}{9 \times 16} = \frac{1}{3}$$

03.

The probability that a bomb dropped from an aeroplane will strike a target is 1 / 5 If four bombs are dropped , find the probability that

a) exactly two will strike the target

b) at least one will strike the target

SOLUTION

4 bombs are dropped , n = 4

for a trial

success – bomb strikes the target

$p = {}^1/_5$, $q = 1 - p = {}^4/_5$

r.v.x = no of successes = 0 , 1, 2, 3, 4

$X \sim B(4, {}^1/_5)$

a) P(exactly two will hit the target)

$= P(x = 2)$

$= {}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2$

$= \frac{6 \times 1 \times 16}{625}$

$= \frac{96}{625}$

b) P(at least one will strike the target)

$= P(x \geq 1)$

$= P(1) + P(2) \ldots.$

$= 1 - P(0)$

$= 1 - {}^4C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^4$

$= 1 - \frac{256}{625}$

$= \frac{369}{625}$

Q6.

(B) Attempt any TWO of the following (08)

01. Find Spearman's Rank Correlation

X	Y	Rx	Ry	d=Rx–Ry	d^2
68	62	4	5	–1	1
64	58	6	7	–1	1
75	68	2.5	3.5	–1	1
50	45	9	10	–1	1
64	81	6	1	5	25
80	60	1	6	–5	25
75	68	2.5	3.5	–1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
				$\Sigma d = 0$	
					$\Sigma d^2 = 72$

CORRECTION FACTORS

Using $\frac{m(m^2 - 1)}{12}$

$tx = \frac{2(2^2 - 1)}{12} + \frac{3(3^2 - 1)}{12}$

$= 0.5 + 2 = 2.5$

$ty = \frac{2(2^2 - 1)}{12} = 0.5$

$\Sigma d^2 \text{corrected} = \Sigma d^2 + tx + ty$

$= 72 + 2.5 + 0.5$

$= 75$

$R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6(75)}{10(99)}$

$= 1 - \frac{5}{11}$

$= \frac{6}{11}$

$= 0.545$

02. Find k if the function f defined by

$$f(x) = kx \quad ; \quad 0 < x < 2$$
$$= 0 \quad ; \quad \text{otherwise}$$

is a p.d.f. of a random variable X

Also find $P(^1/_4 < x < {}^1/_3)$

SOLUTION

$$\int_0^2 kx\,dx = 1$$

$$k\int_0^1 x\,dx = 1$$

$$k\left(\frac{x^2}{2}\right)_0^2 = 1$$

$$k(2) = 1$$

$$k = {}^1/_2$$

$\therefore$ p.d.f. of a random variable X

$$f(x) = \frac{x}{2} \quad ; \quad 0 < x < 2$$
$$= 0 \quad ; \quad \text{otherwise}$$

b) $P(^1/_4 < x < {}^1/_3)$

$$= \int_{1/4}^{1/3} \frac{x}{2}\,dx$$

$$= \left(\frac{x^2}{4}\right)_{1/4}^{1/3}$$

$$= \frac{1}{4}\left\{\left(\frac{1}{9}\right) - \left(\frac{1}{16}\right)\right\}$$

$$= \frac{1}{4}\left(\frac{7}{144}\right)$$

$$= \frac{7}{576}$$

03. A production unit makes special type of metal chips by combining copper and brass . The standard weight of the chip must be at least 5 gm. The basic ingredients copper and brass cost ₹ 8 and ₹ 5 per gm . The durability considerations dictate that the metal chip must not contain more than 4 gm of brass and should contain minimum 2 gm of copper .Find the minimum cost of the metal chip satisfying the above conditions

Let copper input = x gm

Brass input = y gm

CONSTRAINT

1. since standard weight of the chip must be at least 5 gm ;

   $x + y \geq 5$

2. since metal chip must not contain more than 4 gm of brass and should contain minimum 2 gm of copper

   $x \geq 2$; $y \leq 4$

3. since x & y are inputs in gm , cannot be – ve ; $x , y \geq 0$

OBJECTIVE FUNCTION

copper and brass cost ₹ 8 and ₹ 5 per gm

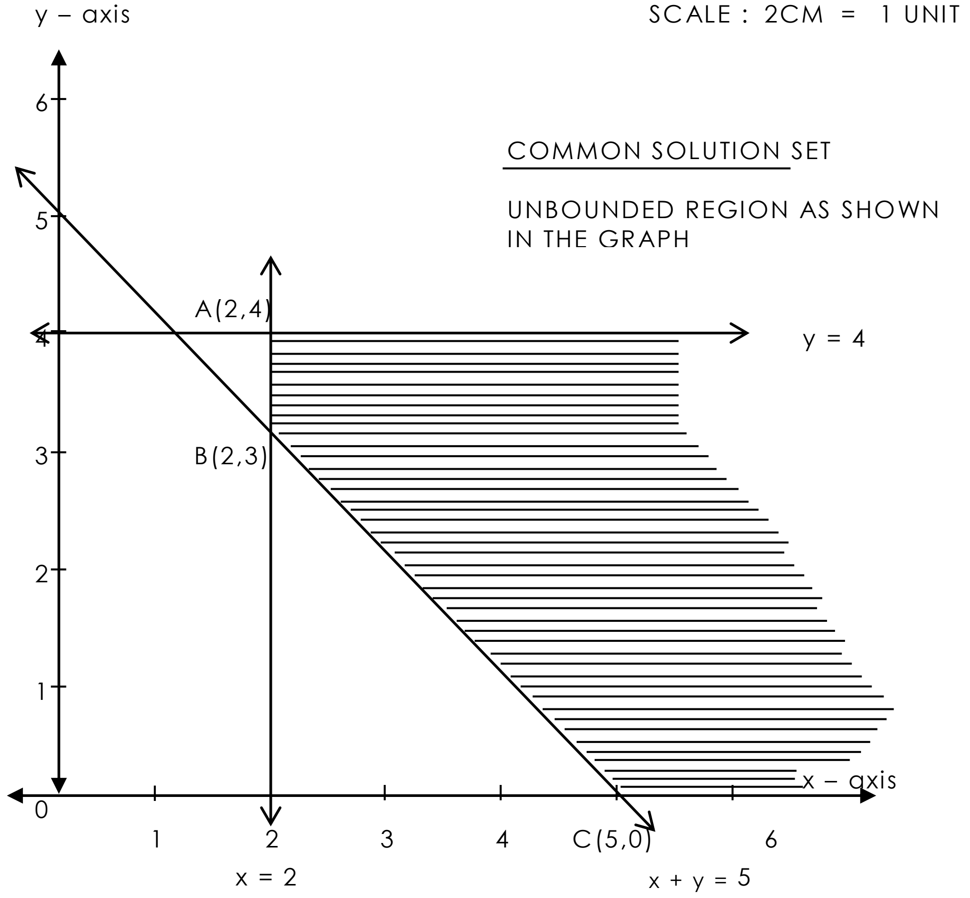
total cost = $8x + 5y$ (in □)

$\therefore$ Minimize $z = 8x + 5y$

LPP MODEL

Minimize $z = 8x + 5y$, Subject to

$x + y \geq 5$; $x \geq 2$; $y \leq 4$; $x , y \geq 0$



CORNERS	Z = 8x + 5y		
A(2,4)	Z = 8(2) + 5(4)	= 16 + 20	= 36
B(2,3)	Z = 8(2) + 5(3)	= 16 + 15	= 31
C(5,0)	Z = 8(5) + 5(0)	= 40 + 0	= 40

OPTIMAL SOLUTION : metal chip should contain 2 gm of copper and 3 gm of brass to keep the cost minimum to ₹ 31